Logarithms for chemists

What's the connection between earthquakes, the acidity of fruit juice and the rates of chemical reactions?

Let's start at the very beginning; a very good place to start (The Sound of Music)

At a very basic level logarithms are just an alternative way of writing down numbers, especially very large or very small numbers. Just to give you an idea, the number 1 000 000 000 can be written as 1×10^9 which is a lot simpler but $\log(1 \times 10^9)$ is, even more simply, 9.

So, how does it work?

The simplest logarithms to understand are the ones based on the number 10. 10 itself can be written as 1×10^1, 100 is 1×10^2, 1000 is 1×10^3 and the logarithms of these numbers are 1, 2 and 3 respectively. These results are written down in this way:

$\log_{10}10 = 1$ $\log_{10}100 = 2$ $\log_{10}1000 = 3$

You don't need a calculator to work out logarithms of numbers like these; you just count the number of zeros after the 1 and you have the logarithm of the number. Alternatively, if your number is in the form 1×10^9 the logarithm of that number is 9.

This is pretty easy so far, but what is the use of it ? Here are two examples of the use of logarithms which you will probably recognise, but you might not realise that logarithms are involved.

Here comes the earthquake …

The first example is the Richter Scale which is used to measure the strength of earthquakes. If an earthquake measures 5 on the Richter Scale, it is 10 times as strong as one measuring 4 on the same scale. The Richter Scale is a logarithmic scale, it reduces large numbers like 10 000 and 100 000 to small numbers like 4 and 5. Notice that the Richter number moves on by 1, but the number itself becomes 10 times larger.

… and here's the fruit juice!

The second example is the pH scale. Fruit juices often have pHs of 4 or 5. A juice with a pH of 4 is ten times as acidic as one with a pH of 5. The pH scale is logarithmic. The pH may change by 1 unit but the acidity changes by a factor of 10.

Inspector Morse might have spotted this … Inspector Clouseau probably wouldn't

You, too, may well have noticed that there is something odd about the second example: it appears that the pH numbers change in the wrong direction. The higher the pH, the *less* acidic the solution is. To appreciate why this is so, we need to see how to work out the logarithms of very small numbers because, although a pH of 4 seems appreciably acidic, the concentration of actual acidic material in a solution with a pH of 4 is very small.

Small is beautiful (and just as simple)

A number such as 0.000 01 can be written in the form 1×10^{-5}. The logarithm of this number is –5. Similarly, the logarithm of 0.0001 is –4. Acidic solutions contain hydrogen ions, $H^+(aq)$, and the pH is measuring the logarithm of the concentration of these ions. Because many people would be confused by negative numbers, the pH is always quoted as a number without the minus sign. Despite this, pH is a logarithmic scale; the acid concentration of a solution with a given pH differs from that of the one with the next pH number not by 1 unit but by a factor of 10.

Calculators are banned (just as well, you probably forgot yours, anyway)

What is the logarithm to the base 10 of each of these numbers ? Write your answers properly in a similar form to this example: $\log_{10}1000 = 3$
The answers are at the end of this article.

(a) 100 (b) 1 000 000 (c) 0.001 (d) 0.000 001

…

Calculators are now unbanned! (So go and find yours!)

To return to the Richter and pH scales; you probably realise that earthquake ratings and pHs are not always integers. We might hear, for example of an earthquake which measures 6.3 on the Richter scale. To arrive at logarithms such as this we shall need some electronic assistance. It is not easy to find the logarithm of a number which is not a simple power or sub-power of 10, so the designers of scientific calculators have built in a function button which does the calculation for you. The way the button is labelled varies with the make of calculator, but it is usually marked ‘log’ , ‘$\log_{10}$’ or ‘lg’. Identify this button and test to see if you have chosen the right one by entering 1000 on the display and pressing your chosen button. The display should change to 3. If it does not do so, you have chosen the wrong button!

Practice makes perfect!

You can now find the logarithms of any number, large or small, simply by entering the number and pressing the ‘log’ button. Try this for the following numbers and check that you get the answers shown.

Number (x)	=	652	1 310 000	0.001 88
$\log_{10}x$	=	2.814	6.117	–2.726

As with any other numbers which result from an operation on the calculator, you should use an appropriate number of significant figures in your answer. Logarithms are very sensitive numbers, in the sense that a small change in the value of a logarithm corresponds to quite a large change in the number itself. The answers have therefore been given to four significant figures instead of the expected three. There is one chemical situation where you should normally not do this. It is very difficult to *measure* pH to a greater precision than one decimal place so there is little point in quoting a *calculated* value to a greater precision than this.

Number 10: the Prime Minister does not have a monopoly!

So, we now know that the logarithm of a number is the power (the technical word is ‘exponent’) of 10 (called the ‘base’) which gives that number. To take an example from the ones you have just done:

$10^{2.814}$ = 652 i.e. 2.814 is the logarithm to the base 10 of 652

The number 10 is the ‘base’ of these logarithms. This is convenient because most of our everyday number work is, quite literally, based on 10.

Doing what comes naturally

Some branches of Mathematics are based, not on 10 but, on another number which is identified by the symbol ‘e’. The value of e, to four significant figures, is 2.718. The symbol ‘e’ honours a mathematician called Euler who investigated the properties of this number. Logarithms which use e as a base are called ‘natural’ logarithms, and in Chemistry it is sometimes necessary to work with formulae which incorporate these e-based logarithms. It is a good thing that we have the benefit of electronics, because you can’t find natural logarithms by inspecting the numbers as you can with some of those based on 10.

More logging

On your scientific calculator you should find a button marked ‘$\log_e$’ or ‘ln’. You use it in a similar way to the ‘log’ button, but you must be careful not to confuse the two. Enter the same numbers as before and find the logarithm to base e of each of them. Check that you get these answers:

Number (x)	652	1 310 000	0.00188
$\log_e x$	6.480	14.09	–6.276

You will notice that the first two logarithms are bigger numbers than the logarithms to base 10. This is because the base e is smaller than 10 so you have to raise it to a higher power to get the same number. The essential idea is the same, however:

$e^{6.480}$ has the same value as $10^{2.814}$, both are 652

So, where do the rates of reaction come in?

When you are studying the way the rate of a reaction depends on the temperature at which it is conducted, you use the Arrhenius equation. This equation incorporates a natural logarithm so you use ‘$\log_e$’ or ‘ln’. You also use natural logarithms when looking at some statistical functions in both kinetics and entropy. When working out pH, however, logarithms to base 10 are used.

Back to the future

Just occasionally you may want to find out what the number is if you know its logarithm. The designers of scientific calculators have thought of that one, too. On your calculator’s keypad you will probably find a button marked ‘Inv’ or ‘Shift’. It may well be in the top left corner of the keypad. The effect of this button is to reverse the function of other function buttons. Try entering 1000 on the display and then press the ‘$\log_{10}$’ button. The display should change to 3; you have just found the logarithm to base 10 of 1000. With the 3 still on the display, press ‘Inv’ followed by ‘$\log_{10}$’ and you should be back to 1000 again. You will need this when you are using the pH of a weak acid to calculate a value for the dissociation constant. If you don’t yet know what a dissociation constant is, you have something to look forward to!

by Alan Furse

Answers:

$\log_{10}100 = 2$ $\log_{10}1\,000\,000 = 6$ $\log_{10}0.001 = –3$ $\log_{10}0.000\,001 = –6$